Unit 6

Revisiting Compositions of 2-Splits and Exploring Equivalence with Standard Units

Partial units are revisited with an eye toward composing repeated 2-splits (partitions). Compositions of partitions provide a way for students to begin to think about the density of fractions and their location on the number line. Fractions, a/b, are quantities representing ***a*** copies of ***b*** congruent partitions. The symbolization a/b corresponds to partitioning the personal unit into ***b*** congruent partitions by splitting (folding) the unit, and then traveling (walking) ***a*** of these partitions. Hence, ¼ unit represents traveling from the origin, 0, to the end of the first of 4 equal partitions of the unit. In contrast, 4/1 represents traveling from the origin 0 to the end of 4 units, because 1 signals that the partition is exactly the same length as the unit. From the perspective of unit iteration, ¼ represents a single iteration or copy of a unit split into 4 congruent partitions, and 4/1 represents 4 iterations (copies) of the unit. Equivalence is represented as the same distance from the origin, no matter the partition (e.g., ½ unit traveled = 2/4 unit traveled = 4/8 unit traveled). The unit concludes with a number line representation of partitioned units and a discussion of how different forms of a/b can result in the same distance traveled.

Materials and Preparation

- 10-12 (1” x 17”) units for each student
- Math journals
- Overhead transparency or individual student handouts of problems 1-5 (listed in this lesson), and in the Appendix

Introduction and Whole-Class Discussion

Introduce the task by asking students to summarize their ideas about splitting units from the last class—or choose several students to read their journal entries that summarized the nature of their learning. Then set the stage for this unit by asking students to consider how they might measure an object’s length using a new unit (1” x 17” that is easily made from 17-inch paper) called an Obama (or choose whatever name you like, but everyone must use that name), if the length of the object is between 1 ¼ OB and 1 ½ OB. What are the advantages to measurement if we all use the same unit? The aim is to **motivate the need to split the unit to promote better approximation.** The use of the same unit allows for easy comparison. This is a principle of standardization. Standards are created by convention.

Have students solve the problems that follow. The teacher can read and write the problems one at a time, or provide the problems on a handout. Have students write their responses on the handout or in their math journals. A whole-class discussion follows after students solve each part of the problem.

Problem 1(a): Take the OB unit and split it into 2 congruent parts by folding. Unfold the OB.

Whole-Class Discussion (after students have attempted the problem):

1. **What do we call the measure of the distance starting at one end of the unit (before we begin to move)?** *(0 OB)*
2. **What do we call the measure of the distance traveled from 0 to the very end of the unit?** *(1 OB)*
3. **How should we label the fold line?** *(1/2 OB)*
4. **What is another name for the very end of the unit?** *(Another name for 1 OB–2 of ½ OB or 2/2 OB)*

Tape the original 1 OB and the newly formed ½ OB onto the board, with the start points aligned vertically.

Ask: **The OB unit is ________ times as long as ½ OB.**

Problem 1(b): Refold a new OB unit so it shows ½ unit. Take another OB unit and beginning at 0, move the folded ½ unit until you travel the length of one personal unit.

1. **How many ½ OB units did you travel?**

Have students answer in the form of "2 half units" rather than just "2."

2. **If we use *of* to stand in for the action of moving the unit or partitioned unit, then ____ of ½ OB = 1 OB. This means: "2 moves of ½ OB travels as far as 1 OB. 2 copies of ½ OB make 1 OB."**

Teacher Note

Iterating a fractional unit until it travels the distance of the original unit establishes equivalence between *n* times as long and the result of *n* iterations of the fractional unit. For example, the unit is 2 times as long as ½ unit, and 2 iterations of ½ unit result in the same length as the unit. Similarly, the original unit is 3 times as long as 1/3 unit, and 3 iterations of 1/3 unit results in the same distance traveled as the unit.

Problem 2(a): Take the OB unit and split it into 2 congruent parts by folding. Then, without unfolding the unit, fold the paper again in the same way. Unfold the paper and look at it.

Whole-Class Discussion (after students have attempted the problem):

1. What should we call the distance starting at one end of the unit and the first-fold line?

Tape the newly formed ¼ OB underneath the previous 2 strips on the board, aligned at the start points.

2. 1 OB is ______ times as long as ¼ OB.

Problem 2(b): Fold a new OB unit so it shows ¼ unit. Using the 1 OB unit beginning at 0, move the ¼ unit until you travel the distance of one personal unit. How many times did it take? *(4)*

1. ______ of ¼ OB = 1 OB

4 copies of ¼ OB make 1 OB and 4 moves of ¼ OB travel as far as 1 OB.

2. Look at the unfolded unit. What should we call the distance between 0 and the second fold line? What else might we call it? Why do you think so?

If I start at 0 and travel ½ OB, or if I start at 0 and travel 2/4 OB, I wind up at exactly the same place. We call this equivalence and show it this way: ½ OB = 2/4 OB.

Teacher Note

The meaning of equivalence is same distance traveled, so that the ending point of the travel is the same, even if it is measured by a different split of the unit. For example, the midpoint of the unit can be reached by starting at 0 and traveling 1 of 1/2, 2 of ¼ (2/4), 4 of 1/8 (4/8), etc. This differs from identity, which implies that even the splits are the same. So ½ is not identical to 2/4 because there are more partitions in 2/4.

Problem 2(b) – continued

3. Look at the unfolded personal unit. What should we call the distance between 0 and the third fold line? Why do you think so?

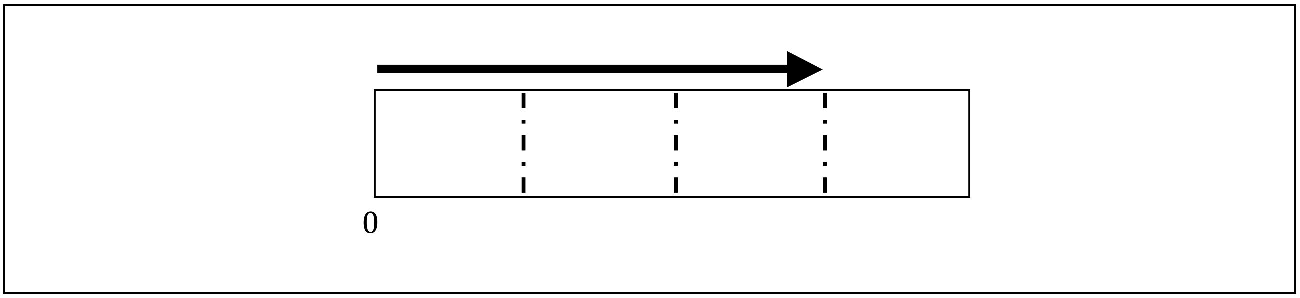


Figure 1. Example of two 2-splits of 1 OB where partitions are labeled with dotted lines. Arrow shows ¾ unit traveled.

4. Look at the unfolded personal unit. What should we call the distance between 0 and the end of the unit? Why do you think so?

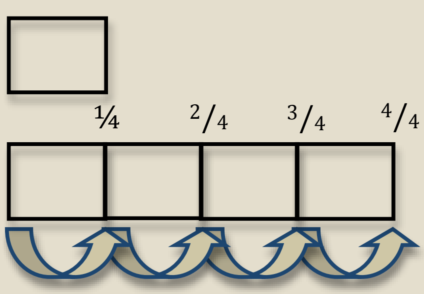
5. Why does 4/4 OB = 1/1 OB? What does 1/1 OB mean?

6. Show, with movement, that 4/4 OB (4 of ¼ OB) travels the same distance as 1/1 OB.

Teacher Note

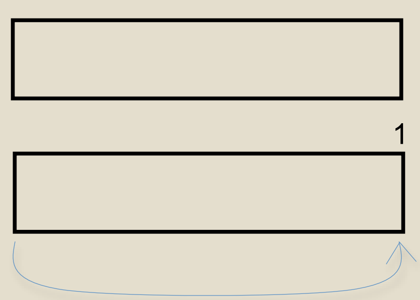
The interpretation of fraction that we are encouraging is that the denominator signals the split of the unit (how many equal partitions of the unit), and the numerator signals how many of that equal-partition have been traveled. So, 1/1 means that the distance traveled is 1 of 1 unit-length (the 1-split leaves the unit length unchanged, meaning that the "origin" of the split is 1). And 4/4 means that the distance traveled is 4, measured in units that are ¼ times as long as the unit-length. 4/4 is equivalent to 1/1 but not identical. That is, 4/4 was obtained first by splitting the unit into 4 equal partitions and then iterating this split-unit 4 times. In contrast, 1/1 was obtained by iterating the unit once. Although we wind up in the same place, the way we traveled there was different. In this nomenclature, ¾ means that the unit was split into 4 equal partitions, each of which was ¼ times as long as the unit-length, and then this split-unit is iterated 3 times.

¼ unit



4 of 1/4

1 unit



1 of 1

Problem 3(a)

Write a prediction for the following question in your math journal (or on the handout):

If we take a personal unit and split it into 2 congruent parts by folding, and then fold it twice more in the same way, what will happen? Why do you think so?

Take a personal unit and split it into 2 congruent parts by folding, and then fold it twice more in the same way, 2 more times. Unfold the paper and look at it. Respond to the following questions in your math journal (with younger children it might be helpful to have questions copied and cut to be pasted into the math journal, rather than copying all the questions):

1. **What did you find out?**
2. **How did the results compare to your prediction?**
3. **Can you explain what happened?**
4. **What should we call the distance starting at one end of the unit and the first fold line?**
5. **Why do you think so?**
6. **How can we write an expression that helps other people know what we did?**

Teacher Note

½ of 1 OB = ½ OB

½ of ½ of 1 OB = ¼ OB

½ of ½ of ½ of 1 OB = 1/8 OB

7. **The personal unit is ______ times as long as the 1/8 personal-unit.**
8. **If we use the word *of* to stand in for moving, _______ of 1/8 unit = 1 personal unit long.**
9. **What does 8/8 = 1/1 mean?**

Problem 3(b): *See Appendix*

Problem 3(c): Challenging Problem — *optional*

Predict what might happen if we split a personal unit by folding it into 2 congruent parts, then fold again in the same way three more times. How might we write about what we have done using numbers and symbols so that others could do the same thing?

Teacher Note

We are aiming to symbolize as follows: It is easy to leave off the "of 1 OB" at the end of the symbolization, but keeping this term on is very important to keep in mind what the original unit is.

½ of ½ of ½ of ½ of 1 OB = 1/16 OB
½ x ½ x ½ x ½ x 1, conventionally, but students may not yet see this as multiplication (we take this up again in a later unit).

Problem 4

Using the number line, show and name:

- a fraction between 0 OB and ½ OB
- a fraction between 0 OB and ¼ OB
- a fraction between ¼ OB and ½ OB
- a fraction between ½ OB and 1 OB
- a fraction between 1 OB and 2 OB

(See Appendix for number lines)

Problem 5

How many different fractions do you think there might be between 0 OB and ¼ OB? Why do you think so?

Student Reflections

Write in your math journals about:

What does equivalence mean? Make a drawing that shows equivalence.

How many fractions do you think there might be between ½ OB and ¾ OB? Why do you think so?

Make a drawing that shows the difference between ¼ OB and 4/1 OB.

Appendix

Name ____ Date________________________

Problem 1

Problem 1(a): Take the OB unit and split it into 2 congruent parts by folding. Unfold the OB.

1. What do we call the measure of the distance starting at one end of the unit (before we begin to move)?

2. What do we call the measure of the distance traveled from 0 to the very end of the unit?

3. How should we label the fold line?

4. What is another name for the very end of the unit?

The OB unit is ________ times as long as ½ OB.

Problem 1(b): Refold a new OB unit so it shows ½ unit. Take another OB unit and beginning at 0, move the folded ½ unit until you travel the length of one personal unit.

1. How many ½ OB units did you travel?

2. If we use *of* to stand in for the action of moving the unit or partitioned unit, then ____ of ½ OB = 1 OB. This means: "2 moves of ½ OB travels as far as 1 OB. 2 copies of ½ OB make 1 OB."

Name ____ Date_____________________

Problem 2

Problem 2(a): Take the OB unit and split it into 2 congruent parts by folding. Then, without unfolding the unit, fold the paper again in the same way. Unfold the paper and look at it.

1. What should we call the distance starting at one end of the unit and the first-fold line?

2. 1 OB is ______ times as long as ¼ OB.

Problem 2(b): Fold a new OB unit so it shows ¼ unit. Using the 1 OB unit beginning at 0, move the ¼ unit until you travel the distance of one personal unit. How many times did it take?

1. ______ of ¼ OB = 1 OB

2. Look at the unfolded unit. What should we call the distance between 0 and the second fold line?

What else might we call it?

Why do you think so?

3. Look at the unfolded personal unit. What should we call the distance between 0 and the third fold line?

Why do you think so?

Name ____ Date________________________

Problem 2(b) – *continued*

4. Look at the unfolded personal unit. What should we call the distance between 0 and the end of the unit?

Why do you think so?

5. Why does 4/4 OB = 1/1 OB?

What does 1/1 OB mean?

6. Show, with movement, that 4/4 OB (4 of ¼ OB) travels the same distance as 1/1 OB.

Name ____ Date________________________

Problem 3

Write a prediction for the following question:

If we take a personal unit and split it into 2 congruent parts by folding, and then fold it twice more in the same way, what will happen? Why do you think so?

Name ____ Date_______________________

Problem 3 – *continued*

Problem 3(a): Take a personal unit and split it into 2 congruent parts by folding, and then fold it twice more in the same way, 2 more times. Unfold the paper and look at it.

1. What did you find out?

2. How did the results compare to your prediction?

3. Can you explain what happened?

4. What should we call the distance starting at one end of the unit and the first fold line?

5. Why do you think so?

Name ____ Date_______________________

Problem 3(a) – *continued*

6. How can we write an expression that helps other people know what we did?

7. The personal unit is _________ times as long as the 1/8 personal-unit.

8. If we use the word *of* to stand in for moving, __________ of 1/8 unit = 1 personal unit long.

9. What does 8/8 = 1/1 mean?

Name ____ Date________________________

Problem 3(b)

If you traveled from 0 to the 1st fold line, you have traveled ____________________ <unit>.

If you traveled from 0 to the 2nd fold line, you have traveled ____________________ <unit>.

If you traveled from 0 to the 3rd fold line, you have traveled ____________________ <unit>.

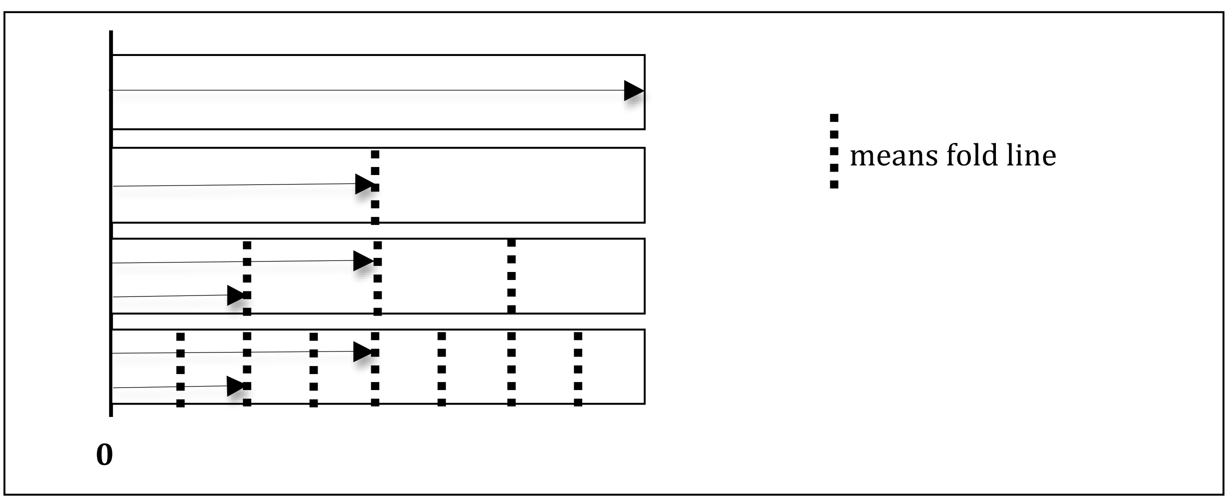
If you traveled from 0 to the 4th fold line, you have traveled ____________________ <unit>.
It could also be called ______________________________ *(be sure to include unit names)*.

If you traveled from 0 to the 5th fold line, you have traveled ____________________ <unit>.

If you traveled from 0 to the 6th fold line, you have traveled ____________________ <unit>.
It could also be called ______________________________ *(be sure to include unit names)*.

If you traveled from 0 to the 7th fold line, you have traveled ____________________ <unit>.

Label the endpoint of each arrow:



Name ____ Date________________________

Problem 3(b)

Distance Traveled			
From 0 to 1st fold line			
From 0 to 2nd fold line	**2/8 unit**	**or ¼ unit**	
From 0 to 3rd fold line			
From 0 to 4th fold line		or	or
From 0 to 5th fold line			
From 0 to 6th fold line		or	
From 0 to 7th fold line			
From 0 to end of unit		or	or

Name ____ Date________________________

Challenge Problem – 3(c)

Predict what might happen if we split a personal unit by folding it into 2 congruent parts, then fold again in the same way three more times.

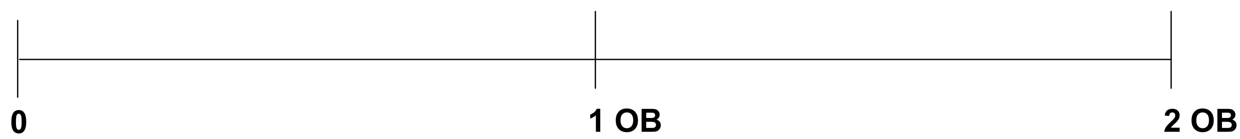
How might we write about what we have done using numbers and symbols so that others could do the same thing?

Name ____ Date_______________________

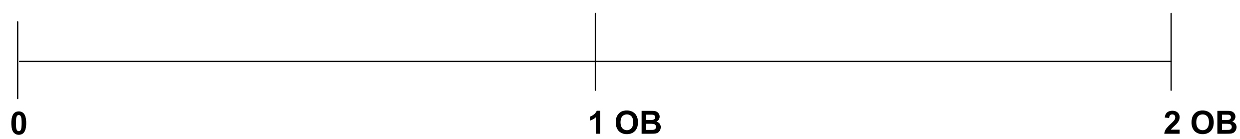
Problem 4

Using the number lines, label:

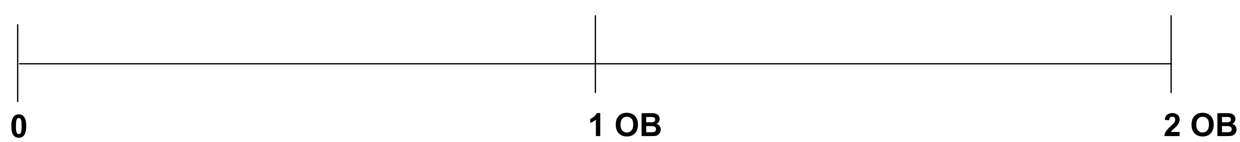
a fraction between 0 OB and ½ OB



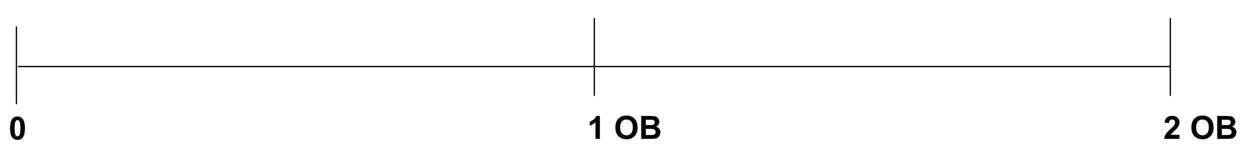
a fraction between 0 OB and ¼ OB



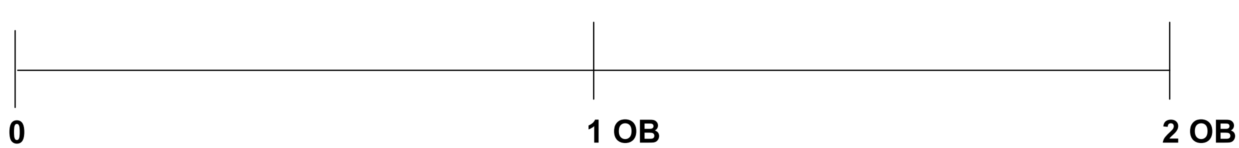
a fraction between ¼ OB and ½ OB



a fraction between ½ OB and 1 OB



a fraction between 1 OB and 2 OB



Name ____ Date________________________

Problem 5

How many different fractions do you think there might be between 0 OB and ¼ OB?

Why do you think so?

Name ____ Date_______________________

Student Reflections

What does equivalence mean? Make a drawing that shows equivalence.

How many fractions do you think there might be between ½ OB and ¾ OB? Why do you think so?

Make a drawing that shows the difference between ¼ OB and 4/1 OB.